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Antioptimization for Comparison of Alternative Structural Models and Damage Detection

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Structural models are usually tested by comparing their responses with those of an actual structure or a very detailed model to a limited number of loads that simulate service loads. However, in some cases (e.g., in the design of some automotive structures), these loads can be arbitrary because there is little information about the actual service loads possibly leading to unreliable results. An antioptimization-based method is proposed for designing numerical or actual experiments to a) test a structural model and b) compare alternative structural models. This method compares a structural model with a reference model or an actual structure under the worst loading case that maximizes the error in the model. The method identifies the loading case that maximizes the difference between the response of two models of the same structure (where one is a reference model) using antioptimization. The proposed antioptimization-based method can be employed also to identify damage in a structure by determining the load that maximizes the difference in the responses of a damaged structure and the intact version of this structure. The proposed method is illustrated by applying it to compare alternate models of an automotive structure and to identify damage in a composite plate.

Nomenclature

D = difference in strain energies of two models
E = Young's modulus
E_{quad} = strain energy stored by the quadrilateral element model
E_{tri} = strain energy stored by the triangular element model
$\boldsymbol{F}$ = load vector
$\boldsymbol{f}$ = vector of load parameters
$\boldsymbol{f}^T$ = transpose of the vector of load parameters
f_x = X direction loading of the orthotropic plate
f_y = Y direction loading of the orthotropic plate
G = shear modulus
$\boldsymbol{K}_1$ = stiffness matrix of model 1
$\boldsymbol{K}_2$ = stiffness matrix of model 2
$\boldsymbol{L}_1$ = flexibility matrix of model 1
$\boldsymbol{L}_2$ = flexibility matrix of model 2
R_e = ratio of strain energies
$\boldsymbol{u}_1$ = displacement vector of model 1
$\boldsymbol{u}_2$ = displacement vector of model 2
γ = Poisson's ratio
$\Delta \boldsymbol{k}$ = difference between the stiffness matrices of zero- and high-mileage cars
Δk_x = stiffness of spring representing change in stiffness of the joint of a car because of high mileage (Δk_x is negative because joint loses stiffness)
θ = fiber orientation angle of the composite material
λ = eigenvalue of the generalized eigenvalue problem
λ_{max} = maximum eigenvalue
σ_x = maximum value of the normal stress in the X direction
σ_y = maximum value of the normal stress in the Y direction
σ_{xy} = maximum value of the shear stress in the X-Y plane

I. Introduction

RECENTLY, composite materials have found widespread applications in many engineering fields. The design of composite plates for high-strength and low-weight applications (as in aerospace) requires the laminates to be relatively thin and free of defects and imperfections. Methods for the modeling of composite structures and for the detection and identification of damage are critical for the successful application of composite structures.

There is no widely accepted analytical or experimental procedure for evaluating the accuracy of a structural model and no widely accepted metric of the accuracy of the model. A model is usually tested by comparing its response to a number of loads, considered representative of service loads, to the response of a reference model or the actual structure. In many practical problems, such as analysis of automotive suspensions and bodies, service loads are not known well, and so somewhat arbitrary loads are used. Moreover, because many load cases are used and the error can vary significantly in each case, it is difficult to decide if a model is acceptable and to compare alternative models.

This paper proposes a procedure for evaluating structural models based on the concept that a system is as good as its weakest link. According to this concept, a model can be evaluated using its maximum error. Antioptimization is used to determine the maximum error. Elishakoff[1,2] coined the term *antioptimization* as a description

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of the process of seeking the most severe design conditions for a structural system. He assumes that uncertainties in loads, material properties, and dimensions are limited to a convex set rather than described by a statistical distribution. Optimization is then used to find the point of the set that represents the most severe combination of conditions.

In this paper we appropriate the term *antioptimization* to the more general process of using optimization to find any worst-case scenario. In particular, we apply it to the process of seeking weaknesses in a model, that is, looking for the conditions under which a model gives the largest errors. This use of antioptimization can help validate a new model, or conversely reveal conditions under which it is not applicable. Thus, antioptimization can be used to establish a process for validating a model by comparing its response to that of a reference model under the most unfavorable loading conditions. Moreover, the maximum error can serve as a measure of the inaccuracy of the model.

Applying antioptimization to a model implies comparison to another model or to experimental results. Gangadharan et al.[3] considered the problem of finding the loading that will maximize the difference in strain energies between two alternative finite element models. They showed that the loading was the solution to a simple eigenvalue problem. Haftka and Kao[4] generalized this to maximizing the ratio of strain energies. They showed that the loading which maximizes the ratio is obtained as a solution of a generalized eigenvalue problem. They also maximized the difference between two composite-laminate failure models by varying the loading, the geometry, and a lamination angle. VanWamelen et al.[5] found the stacking sequence that maximized the difference between two composite laminate failure models and tested that laminate, obtaining results that clearly favored one model.

Damage detection and monitoring of structural systems is becoming critical in many engineering applications.[6] Traditionally, damage detection focused on experimental methods like acoustics (using ultrasonics), thermal field (using infrared imagery), magnetic field sensors, radiographs, and electric fields (using eddy currents) methods. One drawback of these methods is that the location of the damage has to be approximately known and be readily accessed by the equipment. More recently, analytical methods have been used very successfully in many engineering applications.

Analytical damage identification methods[6] can be classified as follows:

1) *Methods that use changes in the natural frequencies.* The shift in the natural frequency because of changes in the structural properties is used to detect damage. Because of the low sensitivity of frequency shifts to damage, the methods require either very precise measurements or large levels of damage. This method is preferrable in a controlled environment where a shift in frequency can be measured precisely as in quality control.[7,8] A principal drawback of this approach is that there is often an insufficient number of frequencies with significant enough changes to determine the location of the damage uniquely.

2) *Methods that use mode-shape changes.* Mode-shape information has been used for the detection of damage without prior use of the finite element model.[9] The mode-shape and mode-shape-slope parameters were simulated for a reduction in stiffness, and the predicted changes were compared to the measured changes to determine the damage location.[10] Graphical comparisons of relative changes in mode shapes proved to be the best way of detecting the damage location when only resonant frequencies and mode shapes were examined. The selection of modes used in the damage identification process is the most important step.[11]

3) *Methods that use mode-shape curvature changes/strain mode-shape changes.* Using mode-shape derivatives (such as curvature) is an alternative to using mode shapes to obtain spatial information about vibration changes. In beams, absolute changes in mode-shape curvature serve as good indicators for damage detection.[12] Parameters that are based on the change in strain mode shape and the change in frequency have also been studied.[13]

4) *Methods based on dynamically measured flexibility.* Dynamically measured flexibility matrix has been used for damage detection. Typically, the damage is detected by comparing the flexibility matrix synthesized using the modes of the damaged structure to the flexibility matrix synthesized using the modes of the undamaged structure or the flexibility matrix from a finite element model.[14] Because of the inverse relationship to the square of the modal frequencies, the measured flexibility matrix is most sensitive to changes in the lower frequency modes of the structure.[15]

5) *Matrix update methods.* Another class of damage identification methods is based on the modification of structural model matrices such as mass, stiffness, and damping to reproduce as closely as possible the measured static or dynamic response from the data. These methods solve for the updated matrices (or perturbations to the nominal model that produce the updated matrices) by forming a constrained optimization problem based on the structural equations of motion, the nominal model, and the measured data.[16] Comparisons of the updated matrices to the original correlated matrices provide an indication of damage and can be used to quantify the location and extent of damage.[17–20]

6) *Nonlinear methods.* Nonlinear elements such as cracks are very difficult to model using finite elements. Modal testing has been used to locate structural nonlinearity using a model update technique with modal data measured at different response levels that are used to localize the nonlinearity.[21] Moreover, nonlinear systems have been simulated using linear techniques. The problem with this approach is that to exactly simulate the nonlinear response one must compute an infinite number of terms.[22]

7) *Neural network-based methods.* Recently, neural networks have been promoted as universal function approximators for functions of arbitrary complexity. The most common neural network is the multilayer perceptron trained by backpropagation.[23] Using networks, damage was located in some structures without error. However, predicting the extent of damage was found to be difficult, and erratic results were obtained.[24]

Most of the methods developed rely on the information provided by the dynamic response characteristic of the structural system.[5] Very few methods employ static loads. The concept of using static loads was used along with neural network methods in Refs. 24–26 and along with vibration tests in Ref. 27 to detect damage.

Sanayei and Onipede[28] used the results of a static test to update the stiffness characteristics of a finite element model. A sensitivity-based, element-level parameter update scheme was used to minimize the error between the applied forces and the forces produced by applying the measured displacements to the model stiffness matrix. The sensitivity matrix was computed analytically. The structural degrees of freedom were partitioned such that the locations of the applied loads and the locations of the measured displacements were completely independent.

Burdisso et al.[29] presented a method for damage identification in automotive joints that compares measured transfer functions of an intact and a damaged structure. This method identifies the joints that have been damaged by finding the part of the car body that has lost stiffness.

Lim[30] presented a systematic method that identifies the damage location and extent accurately when the exact measured modes at every finite element degree of freedom are used. Lim also presented a procedure to perform damage detection with inaccurate, incomplete measured modes based on the stiffness matrix correction technique using submatrices.

Juneja and Haftka,[31] Lee et al.,[32] and Sensmeier et al.[33] used antioptimization along with harmonic response to detect damage. In the past, antioptimization was not used along with static loading data to identify damage. Gangadharan et al.[34] summarized a method that uses simple static loads for model comparison and damage detection.

Most of the preceding methods require a finite element model of the intact structure, which in many cases is not available. Some methods require measuring mode shapes, which is difficult in some cases.

This paper presents an antioptimization-based method for testing structural models and for identifying and detecting damage. This method uses the maximum value of the ratio, or the difference in the strain energies, of two alternative models as a metric of their difference. When one of the models is considered as a reference model, the maximum ratio or maximum difference can be employed as a metric of the accuracy of the model. The paper describes applications of antioptimization for comparing alternative finite element models, using a real life, complex automotive structure as an illustrative example. The paper also demonstrates the use of antioptimization in damage identification.

Section 2 derives the equations for determining the load that maximizes the difference in the responses of two structures or two alternative models of the same structure. Section 3 presents an analytical-experimental procedure for damage detection that is based on the concept of maximizing the difference between the responses of a possibly damaged structure and an intact version of the same structure. Section 4 demonstrates the methods in two examples.

II. Antioptimization of Loading

An important issue in comparison and validation of alternative models and in damage identification is the determination of the loads that should be used. The proposed method uses the load that produces the largest difference in the response of 1) two structures, 2) two alternative structural models, or 3) a structure and the model of this structure. This yields an upper bound on the differences between the behavior of two structural models or structures. In the case where we compare an approximate model to a structure or a reference model, the upper bound on the error of the approximate model can serve as a metric of the fidelity of the model. In the case where we compare a possibly damaged structure and the intact counterpart of this structure, the result also helps to decide if the structures are different and to identify the damage.

The proposed approach makes the ratio of the strain energies of two structures or models extreme, or it maximizes the differences in their strain energies. If the structure is linear, an analytical solution can be found by solving an eigenvalue problem that involves the flexibility matrices of the two structures or models.

Consider two finite element models M_1 and M_2. We seek to antioptimize a vector $\boldsymbol{f}$ of load parameters to maximize the difference between the predictions of the models. The two finite element load vectors $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are assumed to depend linearly on $\boldsymbol{f}$, that is,

$$\boldsymbol{F}_1 = \boldsymbol{A}_1\boldsymbol{f}, \qquad \boldsymbol{F}_2 = \boldsymbol{A}_2\boldsymbol{f} \tag{1}$$

where $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are given matrices. The equations of equilibrium for the two models are

$$\boldsymbol{K}_1\boldsymbol{u}_1 = \boldsymbol{F}_1, \qquad \boldsymbol{K}_2\boldsymbol{u}_2 = \boldsymbol{F}_2 \tag{2}$$

where $\boldsymbol{u}_i$ and $\boldsymbol{K}_i$, $i = 1, 2$ denote the displacement vectors and stiffness matrices associated with the two models (not necessarily of the same order). We employ the ratio R_e of the strain energies as one measure of the differences between the models:

$$R_e = \frac{(\boldsymbol{u}_1^T\boldsymbol{K}_1\boldsymbol{u}_1)}{(\boldsymbol{u}_2^T\boldsymbol{K}_2\boldsymbol{u}_2)} \tag{3}$$

Using Eqs. (1) and (2), we have

$$R_e = \frac{(\boldsymbol{f}^T\boldsymbol{L}_1\boldsymbol{f})}{(\boldsymbol{f}^T\boldsymbol{L}_2\boldsymbol{f})} \tag{4}$$

where $\boldsymbol{L}_1 = \boldsymbol{A}_1^T\boldsymbol{K}_1^{-1}\boldsymbol{A}_1$, $\boldsymbol{L}_2 = \boldsymbol{A}_2^T\boldsymbol{K}_2^{-1}\boldsymbol{A}_2$ are generalized flexibility matrices associated with the two models. In the examples used in this paper, vector $\boldsymbol{f}$ is a common subset of the nodal loads $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. In this case, $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ are ordinary flexibility matrices associated with this subset of the loads. Equation (4) indicates that R_e is a Rayleigh quotient, so that its extreme values are the solution of the generalized eigenvalue problem[3,4]:

$$\boldsymbol{L}_1\boldsymbol{f} - \lambda\boldsymbol{L}_2\boldsymbol{f} = 0 \tag{5}$$

The lowest and highest eigenvalues represent the two extremes of the relative stiffness of the two models.

An alternative measure of the difference between two models is the difference between their strain energies:

$$D = \boldsymbol{u}_1^T\boldsymbol{K}_1\boldsymbol{u}_1 - \boldsymbol{u}_2^T\boldsymbol{K}_2\boldsymbol{u}_2 \tag{6}$$

Using Eqs. (1) and (2), we have

$$D = \boldsymbol{f}^T\boldsymbol{L}_1\boldsymbol{f} - \boldsymbol{f}^T\boldsymbol{L}_2\boldsymbol{f} \tag{7}$$

The smallest and largest eigenvalues of the difference between the generalized flexibility matrices yield the extremes of D. Therefore, the worst loading case is the solution of the following eigenvalue problem[3,4]:

$$(\boldsymbol{L}_1 - \boldsymbol{L}_2)\boldsymbol{f} = \lambda\boldsymbol{f} \tag{8}$$

The worst loading case can be used to identify the most important differences in the models. For example, we can find those directions and locations of the applied forces for which the two models yield significantly different results. This information is difficult to obtain by directly comparing the stiffness matrices of the two models, especially if these matrices are large.

We can also use the maximum ratio of the strain energies, as a measure of the differences between the models [Eqs. (4) and (8)]. This quantity can serve as a practical measure of the difference between models of the same structure or two nominally identical structures. Because this quantity is an upper bound of the ratio of the strain energies of the two models or structures, if it is close to one, we can conclude that the models or structures are similar.

The concept of antioptimization as outlined by Eqs. (5) and (8) was introduced by Haftka and Kao[4] and Gangadharan et al.[3,34]

III. Antioptimization for Damage Detection

A. Procedure for Damage Detection and Identification Using Antioptimization

This section, first, proposes a general procedure for damage detection using the equations for determining the antioptimization load presented in Sec. II. Then it presents a method that uses antioptimization to determine damage in joints in automotive structures because of high mileage.

The loading that maximizes the ratio of the strain energies of two structures can help detect damage and identify its location. Consider an intact structure and a nominally identical structure, which we suspect has been damaged. One can estimate *experimentally* the flexibility matrices of these structures and solve Eq. (5) to find that loading which maximizes the strain-energy ratio (worst case loading). The examination of the worst case loading and comparison of the displacements of the two structures, the strain energies, and their ratio under the worst case loading should help detect damage and identify its location. Indeed, one can reasonably expect that in most of the cases the antioptimization load should be close to the damaged spot.

Consider structures A and B that are identical, except that structure B is suspected to be damaged. The following procedure can be applied to detect and identify damage:

1) Select a few points (say five) in structures A and B. If we have information about the location of damage, we can select these points near this location; otherwise, these points can be evenly distributed in the structure.

2) Estimate experimentally the flexibility matrices corresponding to the points selected in step 1.

3) Determine the worst loading case and the response of the two structures under this worst loading case.

4) Decide whether structure B has been damaged based on the maximum value of the strain-energy ratio.

5) Locate the position and the direction in which the structure has lost stiffness by examining the worst load and the resulting responses of structures A and B. An advantage of this approach is that it does not rely on a detailed model of the intact structure. Instead it uses the measured flexibility matrices of the two structures that correspond to few points.

B. Antioptimization-Based Method for Identifying Damage in Car Joints Due to High Mileage

Car joints lose stiffness because of high mileage. It is important to automotive manufacturers to determine which joints tend to degrade more so that they can improve durability of their products by reinforcing these joints. We propose a method for identifying damage in joints that uses static tests on the same car at zero- and high-mileage stages.

This method consists of two steps. First, it finds the loads that maximize the sensitivity of the static response of a car to damage in

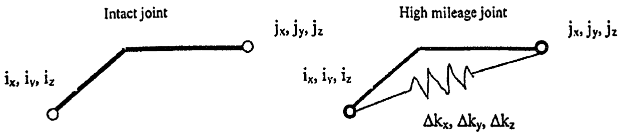

Fig. 1 Damage modeled by three springs Δk_x, Δk_y, and Δk_z connecting two branches of a car joint. The springs represent the change in stiffness in the X, Y, and Z directions.

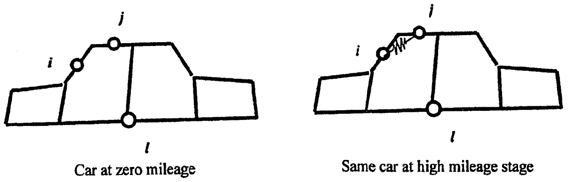

Fig. 2 Schematic showing the comparison of a car at zero mileage and at high mileage.

the joints of the car. Then it finds the loss in stiffness of a joint using the following equation:

{displacement of high-mileage car displacement of zero-mileage car} = [matrix of sensitivities of change in displacement with respect to joint stiffnesses] × {change in joint stiffnesses} (9)

$$L\Delta K =$$

$$\begin{bmatrix} 0 & \Delta k_x(l_{1,ix}-l_{1,jx}) & \Delta k_y(l_{1,iy}-l_{1,jy}) & \Delta k_z(l_{1,iz}-l_{1,jz}) & 0 & -\Delta k_x(l_{1,ix}-l_{1,jx}) & -\Delta k_y(l_{1,iy}-l_{1,jy}) & -\Delta k_z(l_{1,iz}-l_{1,jz}) & 0 \\ 0 & \Delta k_x(l_{2,ix}-l_{2,jx}) & \Delta k_y(l_{2,iy}-l_{2,jy}) & \Delta k_z(l_{2,iz}-l_{1,jz}) & 0 & -\Delta k_x(l_{2,ix}-l_{2,jx}) & -\Delta k_y(l_{2,iy}-l_{2,jy}) & -\Delta k_z(l_{2,iz}-l_{2,jz}) & 0 \\ 0 & \cdots & & \cdots & & & & & \\ 0 & \Delta k_x(l_{n,ix}-l_{n,jx}) & \Delta k_y(l_{n,iy}-l_{n,jy}) & \Delta k_z(l_{n,iz}-l_{n,jz}) & & -\Delta k_x(l_{n,ix}-l_{n,jx}) & -\Delta k_y(l_{n,iy}-l_{n,jy}) & -\Delta k_z(l_{n,iz}-l_{n,jz}) & 0 \end{bmatrix} \tag{17}$$

Damage in a joint is modeled by adding three springs connecting two branches of a joint to the finite element model of a car body. These springs have negative stiffness, and they represent the loss in the stiffness in three directions X, Y, and Z (Figs. 1 and 2). Let $\boldsymbol{F}$ be the force vector and $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ be the vectors of the displacements of the zero- and high-mileage cars, respectively.

In the following, we derive the equations that can be used to estimate Δk_x, Δk_y, and Δk_z for one joint.

We start from the following equations for the displacements of the zero- and high-mileage cars:

$$\boldsymbol{Ku} = \boldsymbol{F} \tag{10}$$

$$(\boldsymbol{K} + \Delta \boldsymbol{K})\tilde{\boldsymbol{u}} = \boldsymbol{F} \tag{11}$$

By subtracting these equations, we obtain

$$(\boldsymbol{K} + \Delta \boldsymbol{K})\tilde{\boldsymbol{u}} - \boldsymbol{Ku} = 0 \tag{12}$$

or

$$\tilde{u} - u = -\boldsymbol{K}^{-1}\Delta \boldsymbol{K}\tilde{\boldsymbol{u}} \tag{13}$$

Equation (13) can be recast as follows:

$$\tilde{u} - u = -\boldsymbol{L}\Delta \boldsymbol{K}\tilde{\boldsymbol{u}} \tag{14}$$

where $\boldsymbol{L}$ is the flexibility matrix of the car at zero mileage.

$$\Delta K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{DK} & 0 & -\boldsymbol{DK} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\boldsymbol{DK} & 0 & \boldsymbol{DK} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \leftarrow \text{Rows: } i_x, i_y, i_z \\ \\ \leftarrow \text{Rows: } j_x, j_y, j_z \end{matrix} \tag{15}$$

Columns: i_x, i_y, i_z Columns: j_x, j_y, j_z

$\boldsymbol{DK}$ is a 3 × 3 submatrix:

$$\boldsymbol{DK} = \begin{bmatrix} \Delta k_x & 0 & 0 \\ 0 & \Delta k_y & 0 \\ 0 & 0 & \Delta k_z \end{bmatrix} \tag{16}$$

where n is the number of displacement measurements and $l_{i,j}$ are the entries of the flexibility matrix of the zero-mileage car. Therefore,

$$\boldsymbol{L}\Delta \boldsymbol{K}\tilde{\boldsymbol{u}} = \begin{Bmatrix} \Delta k_x(l_{1,ix}-l_{1,jx})(\tilde{u}_{ix}-\tilde{u}_{jx}) + \Delta k_y(l_{1,iy}-l_{1,jy})(\tilde{u}_{iy}-\tilde{u}_{jy}) + \Delta k_z(l_{1,iz}-l_{1,jz})(\tilde{u}_{iz}-\tilde{u}_{jz}) \\ \cdots \\ \cdots \end{Bmatrix} \tag{18}$$

Substituting the preceding equation into Eq. (14), we obtain a system of n equations with three unknowns:

$$\tilde{\boldsymbol{u}} - \boldsymbol{u} = -\begin{bmatrix} (l_{1,ix}-l_{1,jx})(\tilde{u}_{ix}-\tilde{u}_{jx}) & (l_{1,iy}-l_{1,jy})(\tilde{u}_{iy}-\tilde{u}_{jy}) & (l_{1,iz}-l_{1,jz})(\tilde{u}_{iz}-\tilde{u}_{jz}) \\ \cdots & & \\ (l_{n,ix}-l_{n,jx})(\tilde{u}_{ix}-\tilde{u}_{jx}) & (l_{n,iy}-l_{n,jy})(\tilde{u}_{iy}-\tilde{u}_{jy}) & (l_{n,iz}-l_{n,jz})(\tilde{u}_{iz}-\tilde{u}_{jz}) \end{bmatrix} \begin{Bmatrix} \Delta k_x \\ \Delta k_y \\ \Delta k_z \end{Bmatrix} \tag{19}$$

We can easily extend Eq. (19) so that it can be applied to problems with m unknown joints. In this case the matrix in Eq. (19) will become a $n \times 3m$ matrix, and the vector that is multiplied by this matrix will have size $3m$. With more equations than unknowns, we find the Δk_x, Δk_y, and Δk_z that minimize the square error, i.e., by regression.

The method presented can be extended easily so that it applies to problems involving dynamic displacements.[28] In this case we should substitute the elements of the receptance matrix for the elements of the flexibility matrix into Eq. (19).

We can use antioptimization to find a loading case that will allow us to determine the unknown changes in the joint stiffnesses with good confidence. Figure 3 summarizes this procedure. First, we select points $l_1, \ldots, l_n$ where loads can be applied and displacements can be measured. For each joint we find the three loads that maximize the difference in, or ratio of, the strain energy of the zero-mileage car and the strain energy of the same car when the stiffness of the joint

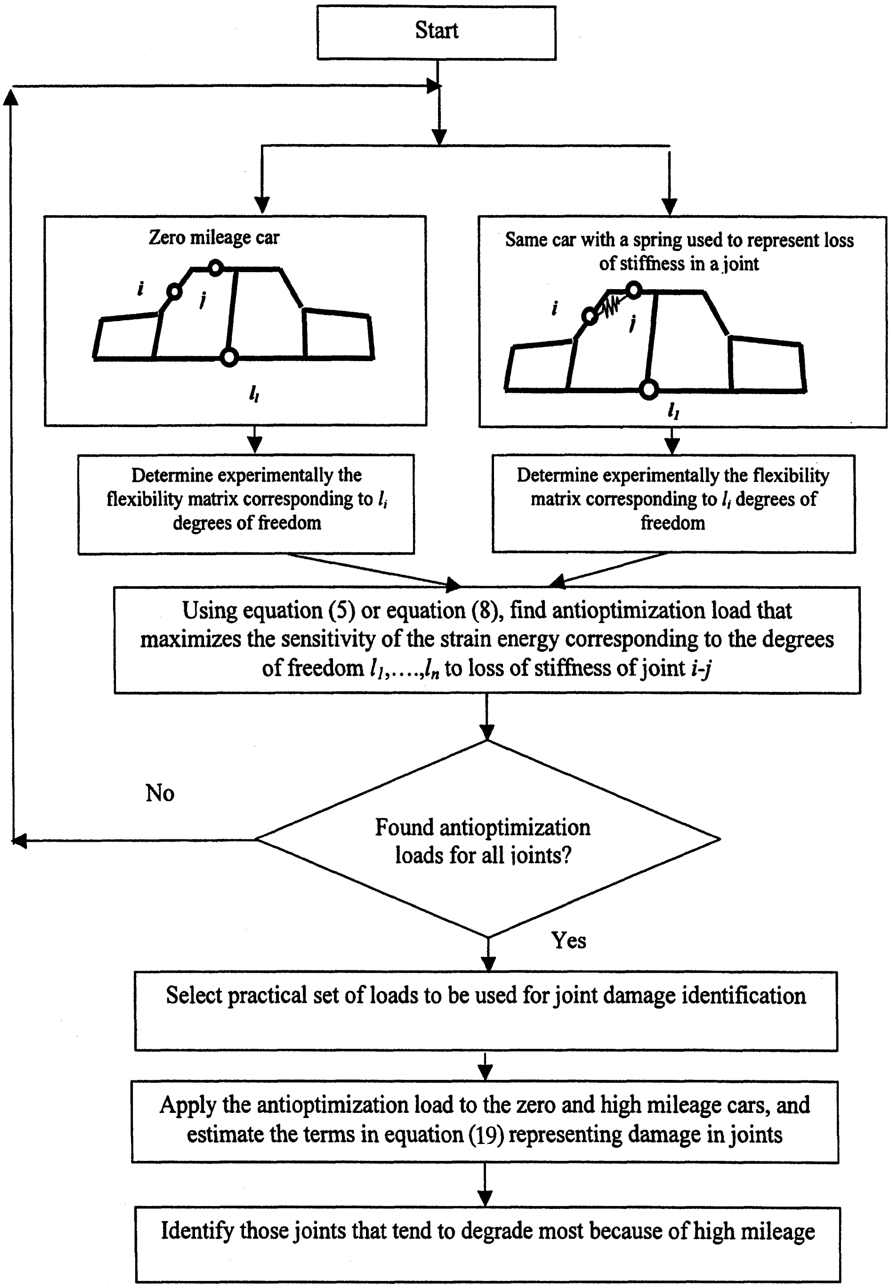

Fig. 3 Method for identification of most vulnerable joints in a car.

changes by Δk_x, Δk_y, and Δk_z, respectively. Using information about these loads, we select a set of loads that is practical to apply to the car at the zero- and high-mileage stages to identify damage in joints.

IV. Illustrative Examples

This section demonstrates the methods for the comparison of alternative structural models and identification of damage using the concepts outlined in the preceding sections. Section IV.A deals with the comparison of alternative structural models whereas Sec. IV.B deals with the identification of damage using the procedure described in Sec. III.A.

A. Comparison of Alternative Structural Models (Automotive Joint Example)

Figure 4 shows a detailed finite element model of a B pillar to rocker joint of an automotive body. The vertical and horizontal members of the joint are called B pillar and rocker, respectively. Lee[35] developed four simplified joint models consisting of torsional springs and rigid elements. The properties of these models are summarized in Table 1.

The parameters of the simplified models are estimated to minimize the differences between the response of simplified and detailed models. Therefore, the simplified models can be almost as accurate as the detailed model under loads that were used in the estimation of parameters. These joint models may not be accurate enough in predicting response under other loads. The engineers or designers need a metric of the accuracy of the simplified joint models before using them for analysis of vehicle structures. The upper bound on the error between the simplified and the detailed models can be such a metric. This quantity can be obtained by considering the worst loads found using antioptimization.

In this study, the parameters of the simplified models were estimated first. Antioptimization was used to compare the simplified joint models with the detailed joint model under the worst load case scenario. The condensed flexibility matrix that corresponds to the six degrees of freedom at the end of the attached beam was considered to compare these models. The worst load vector was found at the end of the beam attached to B pillar (see Fig. 4). The response included static displacements and rotations.

The worst loads that maximize the difference between the strain energies of the simplified and the detailed models, evaluated using Eq. (8), are presented in Table 2. The displacements of these models

Table 1 Properties of the alternative simplified joint models

Model	Property
Simplified model I	The flexibility of a joint is considered using three torsional springs. The rotation centers of these springs coincide with the geometric center of the joint. The orientations of these springs coincide with three orthogonal planes (X_pY_p, Y_pZ_p, Z_pX_p—planes in Fig. 4). Thus, the effect of coupling between rotations in different orthogonal planes is not considered. The parameters in this model are the magnitudes of the torsional springs.
Simplified model II	Same as simplified model I, but the rotation centers of torsional springs do not coincide with the geometric center of a joint. The parameters in this model are the magnitudes and locations of the torsional springs.
Simplified model III	Same as simplified model II, but the coupling effect is considered by allowing the orientations of torsional springs to change arbitrarily. The orientations of three torsional springs are constrained to be mutually orthogonal. The parameters in this model are the magnitudes, locations, and orientations of the torsional springs.
Simplified model IV	Same as simplified model III, but there is no constraint in the orientations of three torsional springs. The parameters in this model are the magnitudes, locations, and orientations of the torsional springs.

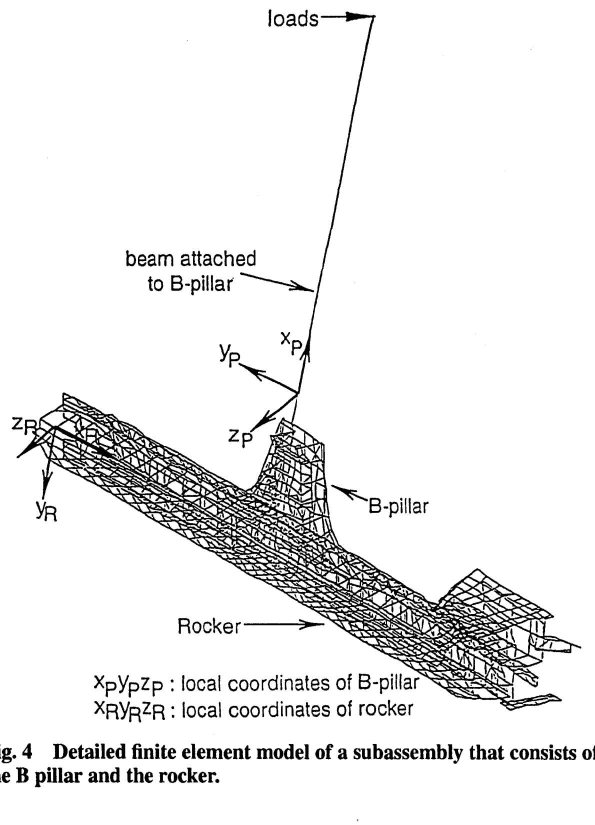

Fig. 4 Detailed finite element model of a subassembly that consists of the B pillar and the rocker.

and the differences of the strain energies under these load vectors are also presented in Table 2.

The displacement components in the X direction of the detailed model do not agree well with any of the simplified models because the axial flexibility was neglected in the four simplified models. However, this discrepancy can be ignored because the portion of strain energy from the force component in the X direction is negligible. For example, in the comparison of simplified model IV with the detailed model, the portion of strain energy in the X direction is only 1.07% of the total strain energy of the detailed model and 0.14% of the total strain energy of simplified model IV.

Under the worst loads the difference between the strain energies of the detailed and the simplified models is 1.85% and 1.79% for simplified model IV and simplified model III, respectively. This difference is small enough so that these simplified models can be used instead of the detailed model under any loads without significant loss of accuracy. If less complicated simplified joint models are used, this difference becomes large. The difference is 7.03% and 18.6% for simplified model II and simplified model I, respectively. The relatively large difference is because the behavior of real joints is not accurately described by these two simplified models. Simplified model I and simplified model II neglect coupling between rotations in different planes. The difference between strain energies of the detailed model and the simplified models is larger for simplified model I than for simplified model II because the former model assumes that the rotation centers of the branches coincide with the center of the joint.

B. Identification of Damage

We can also use the antioptimization procedure described in Sec. III.A to identify damage in a structure. The following experimental procedure can be followed for this purpose:

1) Consider an intact and a damaged structure.

2) Determine experimentally the worst loading case, that is, the load that maximizes the ratio between the strain energies of the two structures.

3) Determine the largest component of the worst load vector. Intuitively, one expects the damage to be in the area most heavily loaded by the worst load. We may also determine the orientation of the damaged fibers from the direction of the worst load.

To demonstrate this approach, we considered a plate consisting of square elements. We simulated the case where there is damage in a fiber by assuming that the Young's modulus of one element in the direction of the fiber is approximately 5% of the moduli of the rest of the elements (Fig. 5). The Young's modulus of the damaged panel and undamaged panels along the X direction (E_x) are taken as 1×10^6 psi and 26.245×10^6 psi, respectively. The Young's modulus of the damaged panel and undamaged panels along the Y direction (E_y), the shear modulus (G_{xy}), and the Poisson's ratio are taken as 1.4935×10^6 psi, 1.0397×10^6 psi, and 0.28, respectively. We considered the loads to be applied at 25 nodes as shown in Fig. 5. Fifty values, which were the components of the load at these points in the X and Y directions, needed to be determined. We found the worst loading case by maximizing the ratio of the strain energies of the damaged and the original plates.

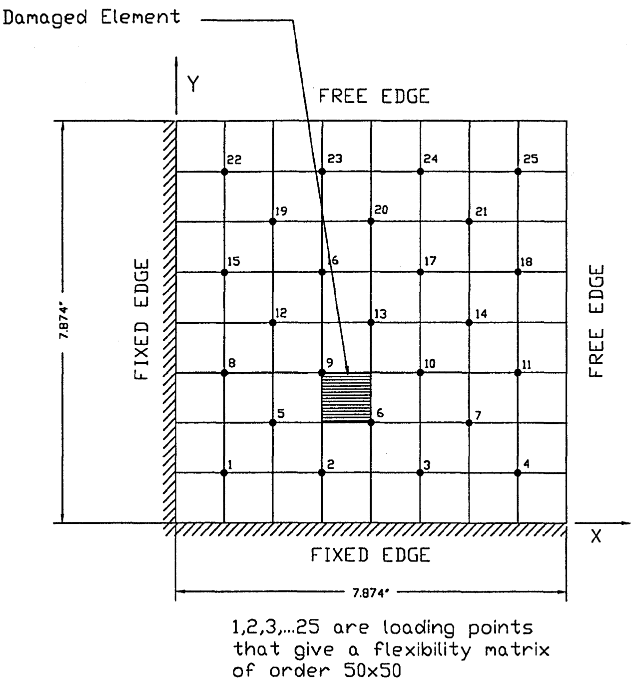

Fig. 5 Plane-stress panel showing the damaged element and loading points.

Table 2 Comparison of simplified models under worst loading case

λ_{max}[a]	Simplified model IV, 5.5929×10^{-4}	Simplified model III, 6.3811×10^{-4}	Simplified model II, 2.4834×10^{-3}	Simplified model I, 1.1086×10^{-2}
		Worst load, f_{worst}		
f_x[b]	−0.7093	−0.6474	−0.1841	−0.0491
f_y	0.1354	0.1036	0.6687	0.1870
f_z	0.6918	0.7551	0.7204	0.9811
M_x[c]	0.0002	0.0002	0.0021	0.0007
M_y	−0.0007	−0.0010	−0.0007	−0.0011
M_z	0.0001	0.0001	0.0006	0.0002
		Displacements under f_{worst} from detailed model		
u_z[d]	-4.5604×10^{-4}	-4.8908×10^{-4}	-5.0526×10^{-4}	-6.1261×10^{-4}
u_y	2.0957×10^{-3}	2.1535×10^{-3}	3.5817×10^{-3}	2.876×10^{-3}
u_z	4.2885×10^{-2}	4.6622×10^{-2}	4.5582×10^{-2}	6.0228×10^{-2}
θ_x[e]	6.1249×10^{-6}	6.7448×10^{-6}	5.55524×10^{-6}	8.6436×10^{-6}
θ_y	-3.7772×10^{-5}	-4.1060×10^{-5}	-4.0203×10^{-5}	-5.3055×10^{-5}
θ_z	-1.9105×10^{-6}	1.9525×10^{-6}	3.3972×10^{-6}	2.6148×10^{-6}
		Displacements under f_{worst} from simplified models		
u_x	-0.5936×10^{-4}	-0.7593×10^{-4}	-0.4801×10^{-4}	-0.6874×10^{-4}
u_y	2.0199×10^{-3}	2.0874×10^{-3}	1.9210×10^{-3}	0.7946×10^{-3}
u_z	4.2499×10^{-2}	4.6140×10^{-2}	4.3793×10^{-2}	4.9351×10^{-2}
θ_x	6.0291×10^{-6}	6.6289×10^{-6}	0.4261×10^{-6}	0.5413×10^{-6}
θ_y	-3.7377×10^{-5}	-4.0404×10^{-5}	-3.8417×10^{-5}	-4.0938×10^{-5}
θ_z	1.8502×10^{-6}	1.8979×10^{-6}	1.9291×10^{-6}	0.7501×10^{-6}
U_{ratio}, %[f]	98.155	98.214	92.969	81.417
ΔU, %	1.845	1.786	7.031	18.583

[a] λ_{max}, largest eigenvalue. [b] f, force component of worst load f_{worst}. [c] M, moment component of worst load f_{worst}. [d] u, translational displacement. [e] θ, rotational displacement. [f] U_{ratio}, ratio of strain energy of the simplified and the detailed models (%). ΔU, difference between strain energies of the simplified and the detailed models (%) normalized by the strain energy of the detailed model.

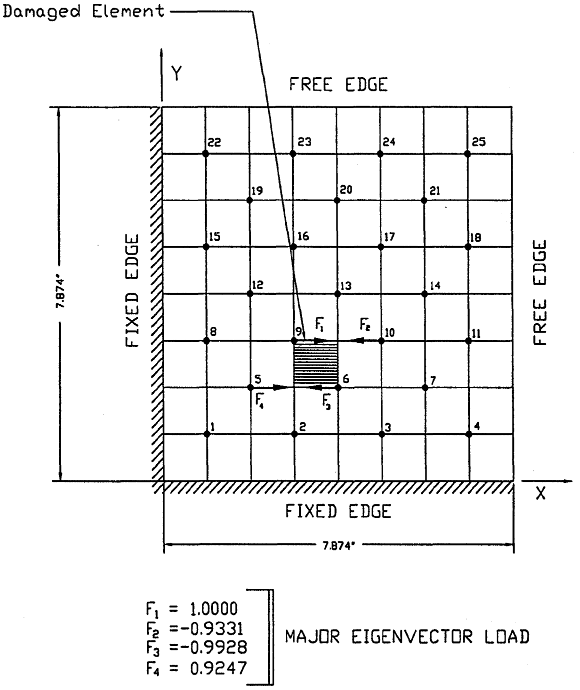

Fig. 6 Eigenvector loading for maximum strain-energy ratio.

The maximum and the minimum ratios of the strain energy were found to be 1.56089 and 0.81412, respectively. The inverse ratio of the minimum eigenvalue $1/\lambda_{min}$ was 1.2283, which was less than the maximum eigenvalue λ_{max}. The worst load consisted of loads applied at the nodes in the vicinity of the damaged element. These loads were parallel to the direction of the damaged fiber as shown in Fig. 6.

This example showed that we can identify the damage in a structure by finding the worst loading case, that is, the load that maximizes the difference between the responses of an intact and the damaged structure. In this case, the worst loading case will be found by a sequence of experiments performed on the two structures. This procedure will be determined using optimization. This is feasible, of course, only if we can repeat the experiments hundreds of times with different loads to perform the optimization.

The proposed approach relies on the hypothesis that the antioptimization load should be located near the damage spot. This hypothesis appears to be reasonable because the antioptimization load should exercise the damaged fibers. However, a large number of examples should be studied to determine the type of problems for which this hypothesis is not true.

One advantage of methods for damage identification that use dynamic measurements is that many measurements can be obtained at the same point by exciting the structure at different frequencies. On the other hand, these methods are not always practical because of the following:

1) They often rely on measurements of the mode shapes, which are difficult and expensive to obtain. In most cases measurements at only a few points on the structure are available.

2) They require an analytical model of the intact structure, which is not always available.[5]

3) The number and location of measurement sensors has not been addressed adequately.[5]

4) It is difficult to discern changes in the structure because of damage to statistical variations in measurements.

The proposed, antioptimization-based approach has the following advantages:

1) It uses experimentally obtained flexibility matrices of the intact and damaged structures, and so it does not rely on an analytical model of a structure.

2) It reveals the difference between the intact and damage structures using that load combination which sharpens the difference in the behaviors of the damaged and intact structures. In many cases this will make it easier to distinguish changes because of damage from statistical variations in measurements.

V. Conclusions

The following are the most important conclusions of this research:

1) An antioptimization-based method to compare two different models representing the same structure was developed and implemented successfully.

2) The method was demonstrated by comparing alternative models of an automotive joint.

3) The proposed method also can be used to identify damage in a structure.

4) The nature of the damage (including the orientation) can be predicted using this antioptimization-based methodology. This concept was illustrated using a simple plane-stress-panel example.

5) A procedure that was specifically developed for identification of damage in automotive joints because of high mileage was presented. This procedure uses antioptimization to determine the load that maximizes the sensitivity of measured displacements to damage in joints.

The following summarizes the example of the automotive joint: In the analysis of automotive structures, simplified models are more practical than detailed models. The accuracy of simplified joint models depends on the loads applied to joints. Using antioptimization, the accuracy of the simplified models was compared under the worst load case that maximizes the difference between the simplified model and the detailed model that is considered as a reference. By considering the worst loads, an upper bound on the error of the simplified models was found. This bound can serve as the metric of the quality of the simplified models.

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References

[1]Elishakoff, I., "On the Uncertainty Triangle," *The Shock and Vibration Digest*, Vol. 22, No. 10, 1990, p. 1.

[2]Elishakoff, I., "Convex Versus Probabilistic Modeling of Uncertainty in Structural Dynamics," *Structural Dynamics: Recent Advances*, edited by M. Petyt, H. W. Wolfe, and C. Mei, Elsevier, London, 1991, pp. 3–21.

[3]Gangadharan, S. N., Nikolaidis, E., and Haftka, R. T., "Probabilistic System Identification of Two Flexible Joint Models," *AIAA Journal*, Vol. 29, No. 8, 1991, pp. 1319–1326.

[4]Haftka, R. T., and Kao, P. J., "The Use of Optimization for Sharpening Differences Between Models," ASME Winter Annual Meeting, Dallas, TX, Nov. 1990.

[5]Van Wamelen, A., Johnson, E. R., and Haftka, R. T., "Optimal Design of Laminated Specimens to Evaluate Competing Failure Criteria," *Proceedings of the American Society for Composites Eighth Technical Conference*, Technomic, Lancaster, PA, 1993, pp. 1045–1055.

[6]Doebling, S. W., Farrar, C. R., Prime, M. B., and Shevitz, D. W., "Damage Identification and Health Monitoring of Structural and Mechanical Systems From Changes in Their Vibration Characteristics: A Literature Review," Los Alamos National Lab., TR LA-13070-MS, Los Alamos, NM, 1996.

[7]Farrar, C. R., Baker, W. E., Bell, T. M., Cone, K. M., Darling, T. W., Duffey, T. A., Eklund, A., and Migliori, A., "Dynamic Characterization and Damage Detection in the I-40 Bridge over the Rio Grande," Los Alamos National Lab. Rept. LA-12767-MS, Los Alamos, NM, 1994.

[8]Migliori, A., Bell, T. M., Dixon, R. D., and Strong, R., "Resonant Ultrasound Non-Destructive Inspection," Los Alamos National Lab. Rept. LA-UR-93-225, Los Alamos, NM, 1993.

[9]West, W. M., "Illustration of the Use of Modal Assurance Criterion to Detect Structural Changes in an Orbiter Test Specimen," U.S. Air Force Conf. on Aircraft Structural Integrity, Presentation 1–6, 1984.

[10]Yuen, M. M. F., "A Numerical Study of the Eigenparameters of a Damaged Cantilever," *Journal of Sound and Vibration*, Vol. 103, pp. 301–310, 1985.

[11]Fox, C. H. J., "The Location of Defects in Structures: A Comparison of the Use of Natural Frequency and Mode Shape Data," *10th International Modal Analysis Conference*, San Diego, CA, Feb. 1992, pp. 522–528.

[12]Pandey, A. K., and Biswas, M., "Damage Detection in Structures Using Changes in Flexibility," *Journal of Sound and Vibration*, Vol. 1, No. 169, 1994, pp. 3–17.

[13]Stubbs, N., Kim, J. T., and Farrar, C. R., "Field Verification of a Nondestructive Damage Localization and Severity Estimation Algorithm," *13th International Modal Analysis Conference*, Nashville, TN, Feb. 1995, pp. 210–218.

[14]Aktan, A. E., Lee, K. L., Chuntavan, C., and Aksel, T., "Modal Testing for Structural Identification and Condition Assessment of Constructed Facilities," *12th International Modal Analysis Conference*, Honolulu, HI, Feb. 1994, pp. 462–468.

[15]Zhang, Z., and Atkan, A. E., "The Damage Indices for Constructed Facilities," *13th International Modal Analysis Conference*, Nashville, TN, Feb. 1995, pp. 1520–1529.

[16]Ojalvo, I. U., and Pilon, D., "Diagnostics for Geometrically Locating Structural Math Model Errors From Modal Test Data," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 29th Structures, Structural Dynamics, and Materials Conference*, Washington, DC, 1988, pp. 1174–1186.

[17]Smith, S. W., and Beattie, C. A., "Model Correlation and Damage Location for Large Space Truss Structures: Secant Method Development and Evaluation," NASA-CR-188102, 1991.

[18]Zimmerman, D. C., and Smith, S. W., "Model Refinement and Damage Location for Intelligent Structures," *Intelligent Structural Systems*, edited by H. S. Tzou and G. L. Anderson, Kluwer Academic, Norwell, MA, 1992, pp. 403–452.

[19]Hemez, F. M., "Theoretical and Experimental Correlation Between Finite Element Models and Modal Tests in the Context of Large Flexible Space Structures," Ph.D. Dissertation, Dept. of Aerospace Engineering Sciences, Univ. of Colorado, Boulder, CO, 1993.

[20]Kaouk, M., "Finite Element Model Adjustment and Damage Detection Using Measured Test Data," Ph.D. Dissertation, Dept. of Aerospace Engineering, Mechanics and Engineering Science, Univ. of Florida, Gainesville, FL, 1993.

[21]Lin, R. M., and Ewins, E. W., "On the Location of Structural Nonlinearity from Modal Testing—Feasibility Study," *Eighth International Modal Analysis Conference*, Kissimmee, FL, Jan.–Feb. 1990, pp. 358–364.

[22]Manson, G., Worden, K., and Tomlinson, G. R., "Pseudo-Fault Induction and Harmonic Suppression in Engineering Structures," *ASME Adaptive Structures and Materials Systems*, AD-Vol. 35, American Society of Mechanical Engineers, New York, 1993, pp. 449–455; Also, *Proceedings of the SPIE*, Vol. 2191, 1994, pp. 499–510.

[23]Bishop, C. M., "Neural Networks and Their Applications," *Review of Scientific Instrumentation*, Vol. 6, No. 65, 1994, pp. 1803–1832.

[24]Kudva, J., Munir, N., and Tan, P., "Damage Detection in Smart Structures Using Neural Networks and Finite Element Analysis," *Active Materials and Adaptive Structures: Proceedings of the ADPE/AIAA/ASME/SPIE Conference*, edited by G. Knowles, IOP, Philadelphia, PA, 1991, pp. 559–562.

[25]Szewczyk, P. Z., and Hajela, P., "Damage Detection in Structures Based on Feature-Sensitive Neural Networks," *ASCE Journal of Computing in Civil Engineering*, Vol. 8, No. 2, 1994, pp. 163–178.

[26]Worden, K., Ball, A., and Tomlinson, G., "Neural Networks for Fault Location," *11th International Modal Analysis Conference*, Kissimmee, FL, Feb. 1993, pp. 47–54.

[27]Zimmerman, D. C., and Simmermacher, T., "Model Refinement and System Health Monitoring Using Data From Multiple Static Loads and Vibration Tests," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 35th Structures, Structural Dynamics, and Materials Conference*, AIAA, Washington, DC, 1994, pp. 325–335.

[28]Sanayei, M., and Onipede, O., "Damage Assessment of Structures Using Static Test Data," *AIAA Journal*, Vol. 7, No. 29, 1991, pp. 1174–1179.

[29]Burdisso, R., Nikolaidis, E., Kuo, E., and Jayasuriya, A., "Nondestructive Estimation of Degradation in Vehicle Joints Due to High Mileage," *Proceedings of the 10th International Conference on Vehicle Structural Dynamics, Society of Automotive Engineers*, 1997, pp. 35–42.

[30]Lim, T. W., "Structural Damage Detection Using Modal Test Data," *AIAA Journal*, Vol. 29, No. 12, 1991, pp. 2271–2274.

[31]Juneja, V., Haftka, R. T., and Cudney, H. H., "Damage Detection and Damage Detectability—Analysis and Experiments," *Journal of Aerospace Engineering*, Vol. 10, No. 4, 1997, pp. 135–143.

[32]Lee, J., Haftka, R. T., Griffin, O. H., Watson, L. T., and Sensmeier, M. D., "Detecting Delamination in a Composite Beam Using Anti-Optimization," *Structural Optimization*, Vol. 8, No. 2/3, 1994, pp. 93–100.

[33]Sensmeier, M. D., Sensharma, K. P., Haftka, R. T., Griffin, O. H., and Watson, L. T., "Experimental Validation of Anti-Optimization Approach for Detecting Delamination Damage," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 36th Structures, Structural Dynamics, and Materials Conference*, Pt. 5, AIAA, Washington, DC, 1995, pp. 3171–3183.

[34]Gangadharan, S. N., Nikolaidis, E., Lee, K., and Haftka, R. T., "Use of Antioptimization to Compare Alternative Structural Models," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 34th Structures, Structural Dynamics, and Materials Conference*, Pt. 1, AIAA, Washington, DC, 1993, pp. 534–543.

[35]Lee, K., "Modeling and Identification of Flexible Joints in Vehicle Structures," Ph.D. Dissertation, Aerospace and Ocean Engineering Dept., Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, 1991.

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